

MECHANICS INTRODUCTION

Text

Contents

Section

- 1 Mathematical Modelling
- 2 Modelling from Data
- 3 Motion and Force
- 4 Newton's Law of Gravitation
- 5 Further Mathematical Modelling
- 6 Units and Dimensions

Mechanics Introduction

1 Mathematical Modelling

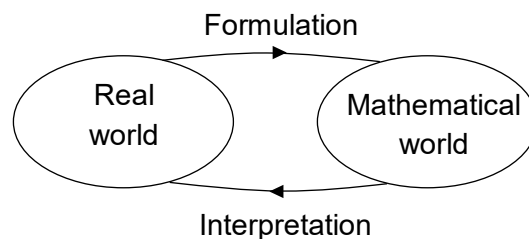
This unit is about the use of mathematics in solving realistic problems. Mathematics occurs in many subjects: not just engineering and physics but also in fields such as economics, biology, epidemiology and transport. Thus it is vital to industry, commerce, government and health. Applying mathematics to such a wide range of subjects needs not only good mathematical problem-solving skills. The mathematician also needs the ability to start with a problem in non-mathematical form, turn it into such a process and then to convert the results of his analysis back once more into non-mathematical language. In between these start and end points, the mathematician must

formulate the problem to allow the use of mathematical analysis,

solve any mathematical problems that have been set up, and then

interpret the solution in the context of the original setting.

This process is called **mathematical modelling** and can be illustrated by the diagram below.



Central to **mathematical modelling** is the representation of the real world problem mathematically. In mechanics the real world problem is commonly represented with a formula. Such a representation is what is meant by a **mathematical model**. The mathematical model could also come in other forms such as a graph, a spreadsheet, a flow chart, and many others.

For example, the formula $s = 15t$, describes s , the distance travelled in metres, in time t , the time in seconds, when travelling at a speed of 15 ms^{-1} . The formula $s = 15t$ is a simple example of a mathematical model.

Broadly speaking, a mathematical model is a relationship between two or more variables. The challenge to the applied mathematician is formulating a good model which accurately describes or represents a given situation. A good model, importantly, can be used to make *predictions* about future behaviour.

The first step in trying to devise a mathematical model for a given situation is to identify the quantities which can be measured and whose values will describe the real situation. In mechanics these quantities might be position, velocity, mass, etc. More generally you might be interested in mortality rates, inflation, interest rates, infection rate of a virus, etc. Such quantities are called **variables**. In the previous example the variables are distance (s) and time (t).

2 Modelling from Data

One of the simplest methods of finding a mathematical model for a given situation is to carry out an appropriate experiment to collect relevant data: from this data a formula relating the variables can be found. This is called **Empirical Modelling**.



Worked Example 1

A motorist records the distance they travel along a motorway at various times. The data are recorded in the table below:

Time in hours	Distance in miles
0	0
1	70
3	210

Produce a model for this data represented by a formula, using the variables s for distance travelled and t for time.



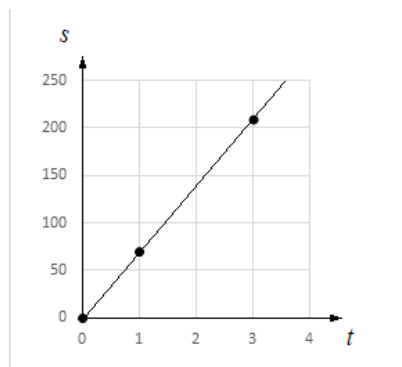
Solution

The data suggest that for every hour travelled, the car covers a distance of 70 miles on average, so you can multiply the number of hours travelled by 70 to obtain the distance travelled in miles. An appropriate formula, therefore, would be $s = 70t$ where t is the time in hours and s is the distance in miles.



Note

The above example is a simple and familiar one, but it illustrates the procedure of taking data and producing a formula from it, going from the real world to the mathematical world. Here the formula is fairly easy to identify, but usually the process of finding a model is much more difficult, commonly a graph is helpful as shown here, with a line of best fit being used to identify the formula.



The graph, table and the formula are all mathematical models and can be used to describe the given situation and to predict what might happen for other variable values. For example, the formula and graph predict that about 245 miles will be travelled in 3.5 hours. Also note that in reality the data will not be so tidy, following a perfect straight line; in practice, with messier data, a line of best fit can be plotted and the formula produced from that. This is a common technique used in mathematical modelling.



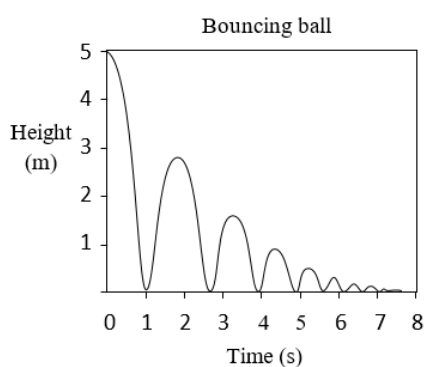
Worked Example 2

A ball is dropped onto a hard floor. A model for the motion of the ball is investigated. Sketch a graph of Height against Time for a ball dropped from 5 metres. You can produce this sketch by doing your own experiment, or you can find videos online which show practical demonstrations of experiments with a bouncing ball.



Solution

A simple experiment of dropping a ball onto a hard surface will illustrate the relationship between the maximum height before each bounce and the maximum height after each bounce. The first rebound height will be less than the original height from which the ball was dropped and then this cycle repeats itself. The graph you produce should look something like this:



The times may be different on your graph, the heights may also be different, but the *pattern* will be the same: each successive height is less than the previous.

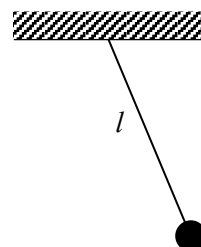
A formula for this behaviour is much more advanced than the previous example, so here the sketch will suffice. However, it will be possible to produce such a formula when you have further knowledge of mathematics, mechanics and mathematical modelling.



Exercise

A simple pendulum consists of a small object attached to a fixed point by a string of length l . As the object swings back and forth the **period** of the pendulum is the time taken for one complete cycle.

- Does the length of the pendulum affect the period?
- Does the mass of the object affect the period?



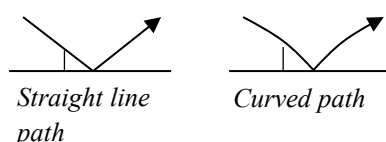
Have a guess at the behaviour first, but you can then experiment or find videos online to answer these questions. Finally, research online or use books to find the formula for the period, T , in terms of the length, l , and mass, m .

3 Motion and Force

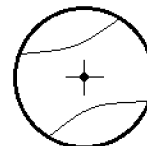
Motion

Mechanics is concerned with the motion of objects and what influences motion. Motion is the change in position and orientation of an object with time. An object's motion can be described simply by its **path**: the route it follows on its journey.

In many cases motion is complicated; take for example the motion of a tennis ball from server to receiver. At first sight it might be thought that the ball moves in a straight line from the server's racket to the ground, before bouncing to the receiver. However, on closer observation the tennis ball is seen to move in a curved path. In fact, the motion of the ball could also involve a swerve and/or a dipping in its path.



In this section the path of an object in simple situations is considered. To do this the object will be treated as a **particle**. This means that the size of the object is ignored and the path of the object is modelled by following some **point** on the object, as shown with the tennis ball on the right; it also allows the spin/rotation of the object to be ignored.



The examples here will only go as far as describing and sketching the path of an object, but later it will be possible to describe the motion mathematically, even producing a formula, thereby being able to predict where an object will be in the future. The mathematical description of motion considers variables of position, velocity, acceleration and time.



Worked Example 1

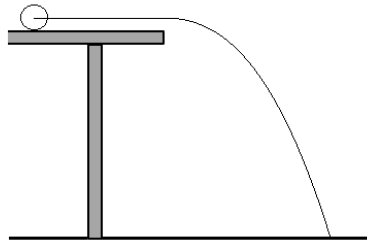
In each of the situations below sketch, or describe, the path of the object:

- A tennis ball is dropped and falls to the ground.
- A golf ball is rolled along a table, then falls to the ground from the table's end.
- A tennis ball is thrown away from a spacecraft, into the blackness of space.
- A satellite orbits the Earth.
- A golf ball is struck into the air and lands a long distance away.



Solution

- a) A straight line, from the point where the ball starts, to the ground.
- b) A straight line along the table then a curved path to the ground:



- c) A straight line away from the astronaut, not ever stopping unless it interacts with another object.
- d) A circle (or ellipse) around planet Earth.
- e) A curve through the air, a *parabola* models this behaviour well:



Force

Force causes a **change in motion**. If an object which is initially at rest (not moving) suddenly starts to move, then something must have caused the motion to begin, it is **force** which has this effect.

You are familiar with forces, and know that if you apply a force to an object it will move. If you ‘push’ an object, such as a pen on a table, then let go, you have applied a force to it, so you will see it continue to slide along the table. Note that it can continue to move even after you let go, and have stopped applying the force.

The principles of force and its effect on motion can be seen more easily in space, where an object is free to move without strong effects of gravity, friction and air resistance. If you apply a force to an object in space it will move, and continue to move, in a straight line away from you, unless something else interacts with it. There are some good films which show the effects of forces in space, such as ‘Gravity’, ‘Apollo 13’ and ‘2001: A Space Odyssey’; there are also good videos online showing real footage from space missions.

A force can also stop an object from moving. When you slide a pen across a table, it is the force of friction which slows down and eventually stops the pen from moving. On an ice rink there is much less friction and an ice puck will continue to move for longer. You could also apply a force to change its direction, changing its path to the left or the right. Finally, you can apply a force to speed it up a little, or you can apply a force to slow it down a little.

So, in summary, a force can

- start motion
- stop motion
- change the direction of motion.
- make an object move faster or slower

These are all examples of forces changing the motion of an object.

When there are no forces acting on an object, it will remain at rest or it will continue to move in a straight line at a constant velocity. This hints at Newton's laws, to be covered in detail in the next unit.



Note

It may be counter-intuitive, but force is **not** required to maintain motion. If you are floating freely in space and you shoot a fire extinguisher, to create a force of thrust, you will begin to move. You will then continue to move, even after you release the fire extinguisher. You will continue to move at a fixed, constant velocity, unless another force is applied to you. The spacecraft Voyager 2 was launched in 1977, it is travelling at over 9 miles per second, is over 11 billion miles from Earth and counting (you can use the NASA website to see how far from us it is now). But this spacecraft is no longer using jets to propel itself: once it was given an initial force of thrust to start its motion, it did not require further force to continue on its path.



Worked Example 2

For the following scenarios, state the effect on the motion of the force being applied.

1. An apple falls off a tree and the force of gravity acts upon it.
2. A biro slides along a long desk, with the force of friction acting on it.
3. A force of thrust on a rocket fires backwards at the rear of the rocket.
4. A person falling to the ground from a great height releases their parachute, producing a force of drag upwards.



Solution

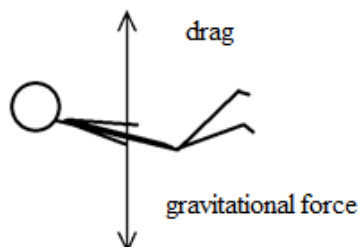
1. It speeds up as it falls to the ground.
2. The force of friction acts against the direction the biro is sliding in, slowing it down.
3. The rocket's speed increases in the forward direction.
4. The person's speed towards the ground begins to reduce, slowing them down.

Multiple Forces and Net Force

When solving Earth-based problems you will usually be dealing with more than one force. When multiple forces act upon an object, the forces combine and can be treated as a single resultant force, called a **net force**.

For example, when a skydiver is falling to the ground from a high altitude, they can obtain a terminal velocity, where they fall at a constant speed towards the ground. From what we know already, this tells us that the net force on the skydiver must be zero, as if it was not, the skydiver would be speeding up, or slowing down. This is because the skydiver is subject to two forces: gravity and drag. Drag is a force that acts upon the skydiver

against the direction they are travelling in, towards the ground, directly opposite to the direction that gravity is acting in. When the skydiver is falling at terminal velocity, the forces of gravity and drag are equal and opposite, cancelling one another out, this results in a net force of zero. Typically this constant velocity is around 120 miles per hour.



Units of Force

The mathematical description of forces leads to a measure for force:

Force is measured in Newtons (N)

The unit is named after Isaac Newton, whose laws will be used in the next unit to solve mechanics problems in a more mathematical way. When you hold a typical smartphone in your hand you will feel a force of a little over 1 Newton, from the weight of the phone. Weight is the first type of force described in the next section.

Types of Force

A number of forces have been mentioned so far: friction, thrust, drag and gravitational force. This section will describe the most common forces that will be encountered in the material that follows. It can be helpful to categorise them as **contact forces** and **non-contact forces**.

Weight (gravitational force)

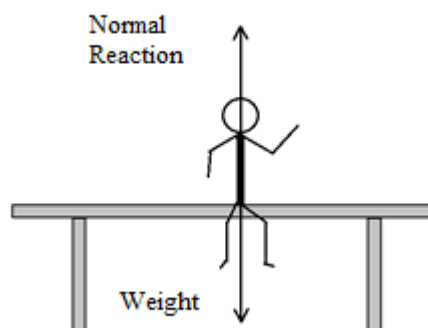
The gravitational force between two objects is the most familiar non-contact force. If you hold a pen in front of you and let it go, it falls to the ground. The force which produced this motion is the force of gravity. Even while an object is in the air and not in contact with any other object, gravity still acts upon it and it continues to speed up. This is why gravitational force is described as a non-contact force.

The gravitational force an object feels is called its **weight**. You can roughly work out the weight of an object on the surface of the Earth, in Newtons, by multiplying its **mass** in kilograms by ten.

All other forces you come across in this section are **contact** forces in which there must be contact between two surfaces for the force to exist. Words such as push, pull, tension, hit, knock, load and friction are used to describe contact forces.

Normal Reaction force

Consider the forces acting on you when you sit on a horizontal table. You have your weight acting on you in a downwards direction. But you are not moving, so there must be another force acting upwards to balance your weight. This force comes from the table and is called the **Normal Reaction** force. These two forces are shown in the diagram on the right.

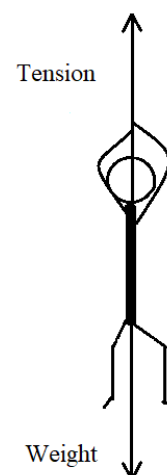


The normal reaction force is a contact force and the direction is at a right angle (or is 'normal') to the table.

Note that whenever two objects are in contact there is a contact force between them. The table exerts a force upwards on you and you exert a force downwards on the table. If the table is not strong enough, it is possible for you to break it.

Tension and Thrust

Consider the forces acting on you when you are hanging from a rope. You have your weight acting on you in a downwards direction. But, again, you are not moving, so there must be another force acting upwards to balance your weight. This force comes from the rope and is called **tension**. These two forces are shown in the diagram on the right.

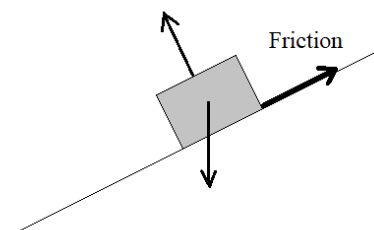


Similar to tension is **thrust**. Whereas tension relates to a material exerting force by pulling you, thrust is a force which *pushes* you, such as a pogo stick, which pushes you up.

Friction

When an object is sliding along a surface, it will tend to slow down. This implies that there is a force acting against it. This force is **friction**. It is another contact force. Friction acts parallel to the surface of contact.

If an object is on a slope, friction acts along the line of the slope, as shown in the diagram on the right. Friction can prevent an object from sliding, or just slow its progress if it is already sliding. The Normal Reaction force and Weight are also shown on the diagram.





Worked Example 4

A book is on a table and is not moving.

- a) Which forces are acting on the book?

The table is now lifted at one end so that it is tilted, so the book is now on a slope. The book still does not move.

- b) Which forces are now acting on the book?



Solution

- a) The book has weight acting on it, in a downwards direction, and the normal reaction force, which is equal and acting upwards.
- b) The book still has weight acting on it, in a downwards direction, with the normal reaction force now acting perpendicular to the table, but now there must be a force of friction too. This frictional force acts along the table, in a direction which points away from the direction that the book is about to slide. The diagram at the bottom of the previous page illustrates these three forces.



Exercises

- Two teams are involved in a tug of war contest. Draw a diagram of the situation using arrows to represent the forces that the rope exerts on each team.
If the teams are equally balanced how do you think that these forces compare
 - in magnitude, and
 - in direction?
- A child is hanging on to a rope tied to the branch of a tree. Draw diagrams to show:
 - the forces acting on the child;
 - the forces on the rope (assume it has zero mass);
 - the force that the rope exerts on the tree.
- While running, an athlete has one foot in contact with the ground. Identify the forces acting
 - on the athlete,
 - on the ground.

4 Newton's Law of Gravitation

In the last section a rough formula was given to find *weight* on the Earth's surface, the force applicable to an object due to its mass: you can multiply the mass of the object in kg by ten to obtain its weight in Newtons.

The weight of an object of mass 1 kg is approximately 10 Newtons.

As a formula this can be written as

$$\text{Weight} = mg, \text{ where } g \sim 10 \text{ ms}^{-2}$$

Experience confirms this result. A bowling ball has a larger mass (~ 7 kg), than a golf ball (~ 0.05 kg). If you hold a golf ball in one hand and a bowling ball in the other hand you will feel the gravitational force of attraction, the weight, of each object pressing down on your hands. The force from the bowling ball, however, is far stronger. So the **force of gravity** of an object depends on the mass of the object.

But this section will give a further example of a well-established mathematical model and through it an improved model for gravity. The model will show that a more accurate value for calculating weight on the Earth's surface is obtained by multiplying the mass by the value 9.81 rather than 10. This section will also introduce a general law for gravity which can be used in problems that are not Earth-based. It will explain why the figure 9.81 is used in more accurate calculations on the surface of the Earth.

You are probably aware that if you were not on the Earth's surface the force of gravity would be different, for example, on a space station, or on the Moon. At this point it is important to understand the difference between mass and weight, since the words are often interchanged and confused in everyday language. Your **mass**, in kg, does not change*, whether you are on the Earth or on the Moon, or elsewhere in space, but your **weight** in Newtons, *does* change depending on where you are. This applies to any object, so if you are standing on the Moon and you are holding the golf ball and the bowling ball, you will find that the weights of the objects are far less than felt on the Earth; little weight is felt from the golf ball and much less weight from the bowling ball.

* unless we take Einstein's special relativity into account, not covered here

mass, in **kg**, is *fixed* and does not change when you are in a different location

weight, in **Newton**s, *changes* depending on where you are

It was Isaac Newton who generalized gravity in his **law of gravitation**; this provides a formula which allows you to calculate the gravitational force between any two objects.

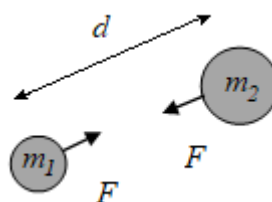
Newton's Universal Law of Gravitation:

Every object in the Universe attracts every other object in the Universe with a force, F , that has magnitude (or size) directly proportional to the masses of the particles, m_1 and m_2 , and inversely proportional to the square of their distance apart, d (the distance between their centres of mass);

$$F = \frac{Gm_1m_2}{d^2}$$

The proportionality constant G is called the gravitational constant and in SI units has the value:

$$G = 6.67 \times 10^{-11} \text{m}^3 \text{s}^{-2} \text{kg}^{-1}.$$

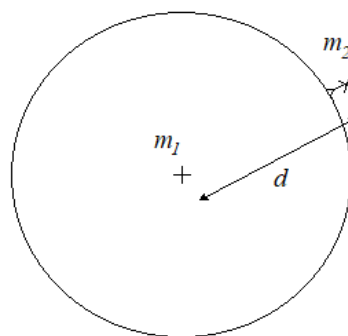


This is a general law, which tells you that any two objects anywhere in the universe will attract one another. It tells you that the gravitational force between two objects is bigger if the objects have a larger mass, and is also dependent on how close the objects are to one another, becoming weaker when the objects are further apart. For you, the largest gravitational force you will feel is towards the ground. This is because the largest object which is close to you is planet Earth, weighing in at 5.98×10^{24} kg. Note that the distance here is the distance between the objects' *centres of mass*; each is a point, which for Earth is at its centre and for a person is inside their body, a little below their navel.

We can now apply this formula to an object of mass m on the Earth's surface. The radius of the Earth is 6.37×10^6 m and its mass is 5.98×10^{24} kg. G is a constant and the value of 6.67×10^{-11} , as given above, is always used. If you apply Newton's law of gravitation to the object, then the force of attraction is

$$F = \frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24}) \times m}{(6.37 \times 10^6)^2}$$

$$\approx 9.81 m \text{ (in Newtons)}$$



So the weight is equal to $9.81 \times \text{mass}$, or roughly $10 \times \text{mass}$ as previously stated.

This agrees with the force of gravity law introduced at the beginning of this section. However, Newton's law of gravitation provides a more general law which can be applied to any two objects anywhere in the Universe!



Exercises

1. Determine the magnitude of the force between a man of mass 70 kg and a woman of mass 60 kg if they are 10 metres apart.
2. An astronaut of mass 75 kg is walking in space at a height of 3×10^7 m measured from the centre of the Earth. What force of gravity does the astronaut experience? How does this compare with the force of gravity on the Earth's surface?
3. The mass of the Moon is 7.38×10^{22} kg and its radius is 1.73×10^6 m. What is the magnitude of the force of gravity on an object of mass m on the Moon's surface?
4. Find the ratio of the force of gravity on an object on the Earth's surface to the force of gravity on the Moon's surface.
5. The distance between the centres of the Earth and Moon is 3.844×10^5 km. Determine how far from the centre of the Earth an astronaut should be, so that the force of gravity of the Earth exactly balances the force of gravity of the Moon.

5 Further Mathematical Modelling

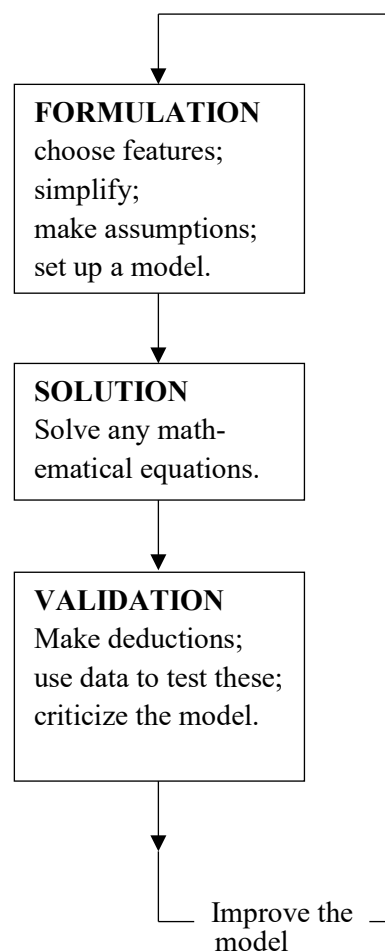
The formulation of the law of gravitation demonstrates another form of mathematical modelling. In the previous section models were formulated by doing experiments and collecting data. From this data the model in the form of a graph and/or an equation takes shape, this is called **empirical modelling**.

Newton's work demonstrates a type of modelling called **theoretical modelling**. You can formulate a model based on an understanding of the physical situation and the important features that affect the situation. Newton proposed that the force between two objects depends on the masses of the objects and the distance between them. He then made three important assumptions:

- the force is an attractive force;
- the force is directly proportional to each mass;
- the force is inversely proportional to the square of the distance.

From these assumptions the model follows.

Appropriate data or experimental activities are then used to validate the model. Notice how this approach uses the data at the end of the activity to test the model and not at the beginning to formulate the model. The diagram on the right summarizes this type of modelling.



6 Units and Dimensions

Throughout this unit physical quantities will be measured in an agreed international system of units called SI units. In this system distances, displacements and positions are measured in metres (m), time in seconds (s) and mass in kilograms (kg).

In mechanics, nearly *all* quantities, somewhat surprisingly, are expressed in terms of basic units of mass, length and time, denoted by the symbols M, L and T.

For example, velocity can be expressed as

$$\frac{L}{T} \text{ or } LT^{-1}$$

and acceleration (ms^{-2}) as

$$\frac{L}{T^2} \text{ or } LT^{-2}$$

The **dimensions** of velocity and acceleration are therefore LT^{-1} and LT^{-2} respectively.

Since SI units are used for the measurements of quantities, we use the following units:

M is measured in kilograms

L is measured in metres

T is measured in seconds

For example, a farmer may be able to carry out a particular task at the rate of 5 hectares per hour. The dimensions of hectares, i.e. an area, are L^2 , so

dimensions of hectares per hour are $L^2 T^{-1}$

Note that numbers like 2, π , 3, etc. have no units, they are said to be *dimensionless*. Some quantities such as force (in Newtons) are not expressed in terms of the common SI units; however, the dimensions can be established:

The Universal Law of Gravitation met in the previous section is given as

$$F = \frac{Gm_1m_2}{d^2}$$

and the proportionality constant G , in SI units, has the value $6.67 \times 10^{-11} m^3 s^{-2} kg^{-1}$.

The dimensions of F are therefore given by combining the dimensions of G , m_1 , m_2 and d^2 .

$$\begin{aligned} \text{So dimensions of } F &= \frac{\frac{L^3}{T^2 M} \times M \times M}{L^2} \\ &= \frac{L^3 M}{T^2 L^2} \\ &= MLT^{-2} \end{aligned}$$

Another way of expressing force, F , is by using Newton's law of motion, $F = ma$, where m is the mass and a the acceleration, this is discussed in the next unit but can be seen to confirm the same result, with Force having dimensions of MLT^{-2} .

As force is in such common use, the unit of the Newton, N, is used for efficiency rather than the unit $kg\ m\ s^{-2}$, based around MLT^{-2} , which would be a bit cumbersome. Many measures in common use have their own units. For example, Pascals (Pa) for pressure rather than the unit $kg\ m^{-1}\ s^{-2}$, as it would be if based on its dimensions, which again would be rather long-winded.

Dimensional Consistency

Equations can be checked for dimensional consistency, in other words the dimensions on both sides of an equation should be the same.



Worked Example 1

The equation for the period of oscillation (the time for one complete swing) of a simple pendulum is

$$Period = 2\pi \sqrt{\frac{l}{g}}$$

where $Period$ = period of oscillation (seconds)

g = acceleration due to gravity (ms^{-2})

l = length of pendulum (m)

Check that the equation is dimensionally consistent.



Solution

The dimension of $Period$ (i.e. the left hand side of the equation) is time, T .

The dimensions of the right hand side of the equation are

$$\sqrt{\frac{L}{LT^{-2}}} = \sqrt{\frac{1}{T^{-2}}} = \sqrt{T^2} = T$$

So the equation is dimensionally consistent.



Worked Example 2

Given that $F = ma$, i.e. force is equal to mass multiplied by acceleration, find the dimensions of force using the right hand side of this equation.



Solution

As acceleration has units LT^{-2} , the right hand side has dimensions MLT^{-2} , as found previously.

Using Dimensions to Produce Formulae

Experimenters sometimes use dimensions to evolve formulae. They create a formula by beginning with an educated guess at it, using measures which would be expected to be related, then they use the principles of balancing dimensions to produce the final formula.

For example, it is known that the frequency, f , of a stretched wire depends on the mass per unit length, m , the length of the vibrating wire, l , and the force, F , used to stretch the wire. The problem is, therefore, to find the relationship between these quantities.

An initial formula is proposed,

$$f = k m^a l^b F^c$$

where k is a constant to be found later and a , b and c are powers to now be determined.

The dimensions of f must be the same as the dimensions of $m^a l^b F^c$.

The dimensions of each quantity are identified:

The frequency means the number of vibrations per second, so its dimensions are T^{-1} .

Mass per unit length m has dimensions ML^{-1}

Length l has dimension L

Force F has dimensions MLT^{-2}

So the dimensions of the proposed formula can be substituted in:

$$T^{-1} = (ML^{-1})^a L^b (MLT^{-2})^c$$

Remove brackets

$$T^{-1} = M^a L^{-a} L^b M^c L^c T^{-2c}$$

Collect powers of each dimension

$$T^{-1} = M^{a+c} \times L^{-a+b+c} \times T^{-2c}$$

Equating the indices for M, L and T on both sides of the equation gives

$$a + c = 0$$

$$-a + b + c = 0$$

$$-2c = -1$$

Hence $a = -\frac{1}{2}$, $b = -1$ and $c = \frac{1}{2}$.

So the formula becomes

$$f = km^{-\frac{1}{2}} l^{-1} F^{\frac{1}{2}}$$

Which can be written more neatly as

$$f = \frac{k}{l} \sqrt{\frac{F}{m}}$$

where k is a constant whose value could be found from experimentation



Exercises

- Which of the following equations are dimensionally consistent?
 - $t = \frac{v-u}{a}$ where t = time, v and u are velocities and a = acceleration.
 - $s = \frac{v^2-u^2}{2a}$ where s = distance, v and u are velocities and a = acceleration.
 - $t = \pi \sqrt{\frac{l^3}{g}}$ where t = period, l = length and g = acceleration.
 - $F = \frac{mu^2}{l} - mg(2 - 3 \cos \theta)$
 where F = force, m = mass, u = velocity, l = length
 and g = acceleration.
- Find the dimensions of each of these quantities which are commonly used in mechanics. Mass is represented by m , velocity by v , acceleration by g , length by h , force by F , time by t and area by A .
 - Kinetic energy E , given by $E = \frac{1}{2}mv^2$
 - Potential energy P , given by $= mgh$
 - Impulse I , given by $I = ft$
 - Pressure P , given by $P = \frac{F}{A}$
- The frequency f of a note given by a wind instrument depends on the length l , the air pressure p and the air density d .
 If f is proportional to $l^x p^y d^z$, find the values of x , y and z .
- The height h , at which a hover mower moves over the ground is thought to depend on the volume of air F which is pumped per second through the fan and on the speed v at which the air escapes from under the mower apron.
 Show that h , F and v could be related by the equation

$$h = k \sqrt{\frac{F}{v}}$$